'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2023

CC8-MATHEMATICS

MULTIVARIATE CALCULUS

(REVISED SYLLABUS 2023 / OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Examine whether $f(x,\ y)=\begin{cases} xy & ; \quad (x,\ y)\neq(0,\ 0) \\ 0 & ; \ \text{if } \ (x,\ y)=(0,\ 0)\end{cases}$ 3

is continuous at the origin.

(b) State Euler's theorem for homogeneous function of two variables. 3

(c) Find the gradient of $\vec{r}+\frac{1}{\vec{r}}$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. 3

(d) Find the volume under the plane $z=8x+6y$ over the region $R=\{(x,\ y):\ 0\leq x\leq 1 \text{ and } 0\leq y\leq 2\}$. 3

(e) Show that for any vector $\vec{a}$, curl $\vec{a}$ is a solenoidal vector. 3

(f) Find the equation of the tangent plane to the surface $z=x^2+y^2$ at the point (1, 2, 5). 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Let $f(x,\ y)=\begin{cases} \dfrac{x^3-y^3}{x^2+y^2} & ; \quad x^2+y^2\neq 0 \\ 0 & ; \ \text{if } \ x^2+y^2=0\end{cases}$. 6

Show that f is continuous at (0, 0) but not differentiable at (0, 0).

3. Find the maximum and minimum values of the function $3x+4y$ on the circle $x^2+y^2=1$. 6

4. Prove that $\iiint\limits_V \overline{\nabla}\times\overline{B}\,dV=\iint\limits_S \hat{n}\times\overline{B}\,dS$, where V is the volume bounded by a closed surface S and $\hat{n}$ is the positive outward drawn normal (unit) to S. 6

5. Prove that $r^n\vec{r}$ is irrotational. Find n when it is solenoidal vector. 6

6. Use Stoke's theorem to evaluate 6

$$\oint_S (\sin z\, dx - \cos x\, dy + \sin y\, dz)$$

where S is the boundary of the rectangle:

$$0 \le x \le \pi\ ,\ 0 \le y \le 1 \text{ and } z = 3$$

7. Evaluate the integral $\iint_R e^{(x+y)/(x-y)}\, dx\, dy$, where R is the trapezoidal region with vertices (1, 0), (2, 0), (0, –2) and (0, –1). 6

GROUP-C

Answer any *two* from the following 12×2 = 24

8. (a) Let $f(x, y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \cos\frac{1}{y} & ;\ x \ne 0,\ y \ne 0 \\ x^2 \sin\frac{1}{x} & ;\ x \ne 0 \\ y^2 \cos\frac{1}{y} & ;\ y \ne 0 \\ 0 & ;\ x = 0 = y \end{cases}$ 2+2+2

Prove that the both partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at (0, 0) but none of them is continuous at (0, 0). Also, examine the differentiability of $f(x, y)$ at (0, 0).

(b) Use Stoke's theorem, prove that 3+3

(i) $\operatorname{curl} \operatorname{grad} \phi = 0$, where ϕ is a scalar function.

(ii) $\operatorname{div} \operatorname{curl} \vec{F} = 0$, where F is a vector field.

9. (a) If E be the region bounded by the circle $x^2 + y^2 - 2ax - 2by = 0$, show that 6

$$\iint_E \sqrt{x(2a-x) + y(2b-y)}\, dx\, dy = \frac{2\pi}{3}(a^2 + b^2)^{3/2}$$

(b) If $f(x, y) = \begin{cases} xy\frac{x^2 - y^2}{x^2 + y^2} & ,\quad (x, y) \ne (0, 0) \\ 0 & ,\ \text{if } (x, y) = (0, 0) \end{cases}$ 6

Show that $f_{xy}(0, 0) \ne f_{yx}(0, 0)$.

10.(a) Find the maximum value of $f(x, y, z) = x^2 y^2 z^2$ subject to the subsidiary condition $x^2 + y^2 + z^2 = c^2$ (x, y, z are positive). 6

(b) Find $\vec{\nabla}(\vec{\nabla} \cdot \vec{A})$, when $\vec{A} = \frac{\vec{r}}{r}$. 6

11.(a) If $\vec{a}$ is a constant vector, then prove that $\operatorname{curl}(\vec{r} \times \vec{a}) = -2\vec{a}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 6

(b) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C (x\, dy - y\, dx)$. Hence find area of the ellipse $x = a\cos\theta$ and $y = a\sin\theta$. 6

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